

TASK 3: Proof Challenges

Evens and Odds

- Every even number can be represented by the expression $2x$ (assuming x is an integer). Explain why this makes sense.
- Every odd number can be represented by the expression $2x + 1$ (assuming x is an integer). Explain why this makes sense.

Proofs

- Prove** that if you multiply any even number with another even number, you get another even number.
- Prove** that if you multiply any even number with any odd number, you get an even number.
- Prove** that if you multiply any odd number with another odd number, you get another odd number.
- Prove** that this pattern will always be true:

$$2^2 = 1^2 + 1 + 2$$

$$3^2 = 2^2 + 2 + 3$$

$$4^2 = 3^2 + 3 + 4$$

$$5^2 = 4^2 + 4 + 5$$

$$6^2 = 5^2 + 5 + 6$$

- Prove** that if you multiply any four consecutive numbers and add 1, you will always get a square number. For example, $5 \times 6 \times 7 \times 8 + 1 = 1,681$ (which is 41^2)

Multiplication Tables

Proofs do not always have to use expressions. They can also be very convincing, logical arguments in sentence form for why something is true.

- How many even numbers are there in a 50×50 multiplication table? Prove how you know.

X	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

- How many multiples of 3 are there in a 50×50 multiplication table? Prove how you know.