Percents, Explained

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Introduction

A **percent** is really just a special way of writing a fraction. When we write a percent, we are really writing a fraction with a hidden denominator of 100. For example:

$27\% = \frac{27}{100}$ $0.5\% = \frac{0.5}{100}$ $\frac{1}{5}\% = \frac{\frac{1}{5}}{100}$ $x\% = \frac{x}{100}$

For this reason, when we convert percents into decimals, we move the decimal two spaces to the left. It's just like converting centimeters into meters. Percents *look* LARGE and decimals *look* small, but they can be used to represent the same quantity. For example:

65% = 0.65	100% = 1	6% = 0.06	2.5% = 0.025
325% = 3.25	8.25% = 0.0825	1% = 0.01	630% = 6.30

Section 1

Finding the New Amount if You Start With the Original Amount

Many percent problems begin with an original amount, corresponding to 100%. Then there's a new amount, corresponding to a smaller or higher percentage. The goal is to find the new amount using either proportions or decimals.

Example 1 (Plain Math): Find 30% of 60
60 is the original amount, so it goes with 100%. The goal is to find the smaller amount, which goes with 30%. **Match two quantities by putting them across from each other. Two matching quantities should never be diagonal.**

$$\frac{x}{60} = \frac{30\%}{100\%}$$

$$x = 18$$

The same problem can be solved using decimals. The word "of" usually means multiplication of decimals, so we can solve using the calculation:

"30%" "of" "60"

$$0.30 \cdot 60 = 18$$

Example 2 (Tax/Tip): Find the 8.25% tax on a $1,200 laptop.
This is virtually the same as example 1. The $1,200 is the original price, so it goes with 100%. The goal is to find the modified amount, which goes with 8.25%.

$$\frac{x}{1200} = \frac{8.25\%}{100\%}$$

$$x = \$99$$

Like before, we can also solve this using multiplication. Just remember to convert the percent into a decimal by moving the decimal two spaces to the left. Single-digit percents do look like very small decimals, so don't mistake 8.25% with 8.25 (they're not the same).

"8.25%" "of" "1,200"

$$0.0825 \cdot 1200 = \$99$$

Example 3 (Tax/Tip Total): Find the total after an 8.25% tax on a $1,200 laptop
This problem comes from Example 2. It can be solved by simply adding the $99 tax with the $1,200 original price to get $1,299.

However, you can also interpret this problem the following way: $1,200 is the original price, so it goes with 100%. The new total price is 8.25% greater, so it goes with 108.25%. Let that sink in.

$$\frac{x}{1200} = \frac{108.25\%}{100\%}$$

$$x = \$1{,}299$$

Of course, we can also convert 108.25% into a decimal and multiply:

“108.25%” “of” “1200”

$$1.0825 \cdot 1200 = \$1{,}299$$

Example 4 (Discount): Find the new price of a $1,200 laptop that receives a 15% off discount This problem can be solved in a similar manner as Example 3. Like before, $1,200 is the original price (100%) and the new price is 15% less than that. In this sense, the new price corresponds to 85% (make sure you understand why).

Proportions!

$$\frac{x}{1200} = \frac{85\%}{100\%}$$

$$x = \$1{,}020$$

Decimals!

“85%” “of” “1200”

$$0.85 \cdot 1200 = \$1{,}299$$

Practice Problems:

1) Find 25% of 75

2) Find the 45% tax on a $1,300 laptop

3) What is 99% of 200?

4) Find 46% of 250

5) Find 40% of 450 (*Sun Min C.*)

6) I have 20 apples. 60% are eaten. How many do I have left?

7) Bill went to a pizza parlor and bought a pie for $16. Luckily, the day he visited was "Pizza Wednesday" and he got a 20% off discount. How much did he pay for his meal (without any tax or tip)?

8) A bike costs $900 with a 6% tax. If Alex has $910, can she buy the bike?

9) Find the total after a 7% tax and a $4 tip on a $150 meal

10) Emma bought a TV for $400. She got a 21% off discount on the TV. How much did she pay at the register?

Solutions:

1) 18.75
2) $585
3) 198
4) 115
5) 180
6) 8 apples
7) $12.80
8) No. The bike will cost $954 including tax and $910 is $44 less
9) $164.50
10) $316

Section 2

<u>Finding the Original Amount if You Start With the New Amount</u>

This is the reverse of the first type of problem. You begin with the new amount, which might be larger or smaller than the original amount. You have to figure out the original amount, which corresponds to 100%. *Read every problem carefully to make sure you know whether you're being told the original or the new amount.*

These types of problems come in different flavors like before (plain math, tax/tip, and discount).

Example 1 (Plain Math): 60 is 75% of what number?
The problem indicates that 60 goes with 75%. Logically, x must go with 100%. Before even solving, ask yourself whether x will be bigger or smaller than 60.

Proportion Method

$$\frac{60}{x} = \frac{75\%}{100\%}$$

$$x = 80$$

Decimal Method: Using a decimal to find an original amount takes a bit of logic. Suppose you knew x. Since 60 is 75% of x, then you'd multiply 75% times x and get 60. You can write this as an equation then solve for x:

"60" "is" "75%" "of" "x"

$$60 = 0.75 \cdot x$$

$$80 = x$$

(I got this by dividing both sides by 0.75)

Example 2 (Plain Math): 120 is 105.25% of what number?
120 goes with 105.25%. The original number corresponds to 100%, so the original number must be *smaller* than 120.

Don't be intimidated by decimals in percents. Treat them like any other number. Round your answer to the nearest hundredth as needed (or follow the directions if different).

Proportions:

$$\frac{120}{x} = \frac{105.25\%}{100\%}$$

$$x = 114.01$$

Decimals:

"120" "is" "105.25%" "of" "x"

$$120 = 1.0525 \cdot x$$

$$114.01 = x$$

(I got this by dividing both sides by 105.25)

Example 3 (Tax/Tip): The price of a bag after 5.25% taxes is \$58.94. Find the original price. The original price is 100% but it's unknown. However, we do know that after we add on 5.25%, we get \$58.94. This means \$58.94 goes with 105.25% (make sure you understand why). Does this problem seem familiar? It should because it's just like Example 2.

Here's the proportion:

$$\frac{58.94}{x} = \frac{105.25\%}{100\%}$$

$$x = \$56$$

Here's the decimal method:

"58.94" "is" "105.25%" "of" "x"

$$58.94 = 1.0525 \cdot x$$

$$\$56 = x$$

(I got this by dividing both sides by 1.0525)

Example 4 (Discount): A meal after a 20% discount was \$34. Find the original bill. It would be easy to rush through the problem and believe that \$34 corresponds to 20%. However, a little more thinking would show that \$34 actually goes with 80% because it represents what remains after 20% is removed from the original price.

Proportion:

$$\frac{34}{x} = \frac{80\%}{100\%}$$

$$x = \$42.50$$

Decimals:

"34" "is" "80%" "of" "x"

$$34 = 0.80 \cdot x$$

$$\$42.50 = x$$

(I got this by dividing both sides by 0.80)

Practice Problems

1) 60 is 50% of what number?
2) 60 is 70% of what number?
3) 11,375 is 80% of what number?
4) If 82 is 3% of a number, what is that number?
5) 70 is 45% of what number?
6) After a 25% discount, the total cost of 2 shirts and 1 pair of pants was $35. How much would the clothes have been without the discount?
7) John bought a toy for $46.75, which included a discount of 15%. What was the original price of the toy?
8) The price of a car is $12,000 after 2% tax is applied. What was the original price?
9) After paying a 15% tip, Bob pays $86 at the local pizzeria. What was the original price?
10) Robin tried to pay $30 for a video game. But because he did his math wrong, he was told that this was only 85% of the total payment. How much should Robin pay?

Solutions

1) 120
2) 85.71
3) 14,218.75
4) 2,733.33
5) 155.56
6) $46.67
7) $55
8) $11,764.71
9) $74.78
10) $35.29

Section 3

Finding the Percent if You Start With the Original and New Amounts

This is the last type of problem. The goal is to find the *percent* if you know the original and new amounts. Like before, it comes in three flavors: plain math, tax/tip, and discounts.

Example 1 (Plain Math): 12 is what percent of 32?
The problem suggests that 12 is a part of 32. In other words, 32 is the original amount (100%) and 12 corresponds to some unknown percent. This unknown percent will be less than 100%.

This can be solved with a proportion the same way the other two types of problems were solved.

$$\frac{12}{32} = \frac{x\%}{100\%}$$

$$x = 37.5\%$$

Example 2 (Plain Math): Convert $\frac{12}{32}$ into a percent.
This example uses the same numbers as Example 1 because they are the same problem. The denominator represents the original amount (100%) and the numerator represents the new amount. Therefore, one method to convert a fraction into a percent is to use a proportion. The goal is to turn the fraction into one with a denominator of 100 (after all, that's the definition of a percent).

Another method is to use long division, and then convert the decimal into a percent.

		0	0.	3	7	5
3	2	1	2.	0	0	0
	−	0				
		1	2			
	−		0			
		1	2	0		
	−		9	6		
			2	4	0	
		−	2	2	4	
				1	6	0
			−	1	6	0
						0

0.375 can be converted into 37.5%

Example 3 (Plain Math): 18 is what percent of 15?
Don't get into the habit of looking at the numbers and thinking you know what the question is asking. In this example, the word "of 15" means that 15 is the *original amount* and 18 is the new amount.

Weird, right? But that just means the percent you'll find is greater than 100% since you're taking a smaller number (15) and turning it into a bigger one (18).

Proportion:

$$\frac{18}{15} = \frac{x\%}{100\%}$$

$$x = 120\%$$

Long Division Method: Remember, long division only gives you a decimal. To convert to a percent, move the decimal place two spaces to the right (*percents are "bigger")*

		0	1.	2
1	5	1	8.	0
	−	0		
		1	8	
	−	1	5	
			3	0
		−	3	0
				0

Example 4 (Tax/Tip): Find the tax rate if socks were originally \$4.50 but you paid \$4.77. The \$4.50 matches with 100% and the \$4.77 matches with the higher unknown percent. But this higher percent is not the tax rate itself. Instead, the tax rate is found by subtracting away 100% (can you figure out why).

With a proportion:

$$\frac{4.77}{4.50} = \frac{x\%}{100\%}$$

$$x = 106\%$$

Tax Rate = 6%

It's also possible to solve this using our percent change equation:

$$\frac{|Original - New|}{Original}$$

The $|Original - New|$ is the symbol for "absolute value." It means ignore any negatives if they come up.

Lastly, don't forget the equation gives a decimal. You must convert it into a percent.

$$\frac{|4.50 - 4.77|}{4.50} = 0.06$$

0.06 can be converted into 6%

Example 5 (Discount): A $32 videogame is on sale for $20 on Amazon. Find the discount. The $32 is the original price and the $20 is the new sale price. Beware: The percent you get is not the discount. You must subtract with 100% to see how much percent is actually being removed. The removed amount is the discount.

Proportions:

$$\frac{20}{32} = \frac{x\%}{100\%}$$

$$x = 62.5\%$$

Tax Rate = 37.5%

Practice Problems

1) 30 is what percent of 120?
2) 37 is what percent of 56?
3) 15 is what percent of 24?
4) 20 is what percent of 15?
5) Convert $\frac{56}{234}$ into a percent without the use of a calculator (rounded to the nearest thousandths)
6) Tim has $100, he suddenly has a total of $150. What is percent change from $100 to $150?
7) Ravi got 18 marks out of 20 in his Math Test? How much is that in percentage?
8) A video game that was originally $40 is now $38 dollars. What was the percent discount?
9) Subject Z buys a generator for $1,582, when it was originally $1,600. What was the discount Subject Z got on the electric generator?
10) Joe had $79664 in his bank account. Then, someone stole his credit card number, and withdrew money, but luckily Joe froze his credit card before the robber could take any more money. Joe was left with $55647 left. What was the percent change from his original balance to now?

Solutions

1) 25%
2) 66.07%
3) 62.50%
4) 133.33%
5) 23.93%
6) 50%
7) 90%
8) 5%
9) 1.125%
10) 30.15%

Section 4

A Note On Equivalent Ratios

You might be wondering why I haven't mentioned any other ways to solve these problems. And in case you didn't realize it, there *are* other ways to solve all of the examples in the previous sections.

The biggest omission has been equivalent ratios. Equivalent ratios are *the* universal method and can be applied to any problem. Some problems lend themselves naturally to equivalent ratios. Some don't.

Example 1: 60 is 75% of what number?
Yes, you can create a proportion. But you can also realize that dividing both 60 and 75% by 3 tells you that 20 is 25%. Then multiplying both by 4 tells you 80 is 100%. This can be shown with a diagram:

$$\frac{60}{75\%} \to \frac{20}{25\%} \to \frac{80}{100\%}$$

The original number is 80

Example 2: A $32 videogame is on sale for $20 on Amazon. Find the discount.
Discounts involve percents. In a percent, the original amount corresponds to 100. So let's turn the $32 videogame into 100%. Then let's see the new percent. The difference will tell you the discount.

$$\frac{20}{32} \to \frac{0.625\%}{1\%} \to \frac{62.5\%}{100\%}$$

The discount is the difference between 100 and 62.5. It's 37.5%

Example 3: Find 30% of 60
We start with $\frac{60}{100\%}$ then convert that into a ratio over 30%. Well how much is 10%? It's 6. So 30% much be 18. Diagram:

$$\frac{60}{100\%} \to \frac{6}{10\%} \to \frac{18}{30\%}$$

So 30% of 60 is 18

I could go on, but I'll stop. It should be evident that there is no formal procedure for equivalent ratios. It takes experience and creativity to use them. And yet they can be quite rewarding to figure out. They're like interesting puzzles. They're a good way to check your work. And they can often be easier than using a proportion or decimal.

Practice Problems

1) Solve every practice problem in Sections 1-3 using equivalent ratios.

Section 5

Challenge and Other Problems

The examples and problems in this chapter have focused on plain math, tax, tip, and discount. However, percent and ratio problems cover a wide range of contexts, from the percent of blue-eyed students in a class to the ratio of two-eyed fish to three-eyed fish in a lake.

Even though the types of problems may vary, they can often be related to the types of problems covered in this chapter. Any quantity greater than the original amount is a tax/tip in disguise. Likewise, any quantity less than the original amount can be regarded as a discount.

In this section, I've included a collection of challenging problems and any other puzzles or problems I happen to find interesting. Please email me at melvinmperalta@gmail.com if you have any challenging problems that you think could be added to this section.

A Collection of Challenging Problems

1) A $480 TV was put on sale for 30% off. It didn't sell so the price was lowered an additional percent off the sale price, making the new price $285.60. What was the second percent discount that was given?

2) Working alone, Jamie can mow her lawn in 75 minutes. If Bob helps her, then the two can mow the lawn in 30 minutes. How long does it take Bob to mow the lawn alone?

3) Four short-order cooks can make 24 omelets in 10 minutes. If a diner gets a to-go order for 90 omelets that needs to be ready in 15 minutes, then how many cooks do they need to complete the order on time?

4) The ratio of Jim's money to Peter's money was 4:7 at first. After Jim spent half of his money and Peter spent $60, Peter had twice as much money as Jim. How much money did Jim have at first?

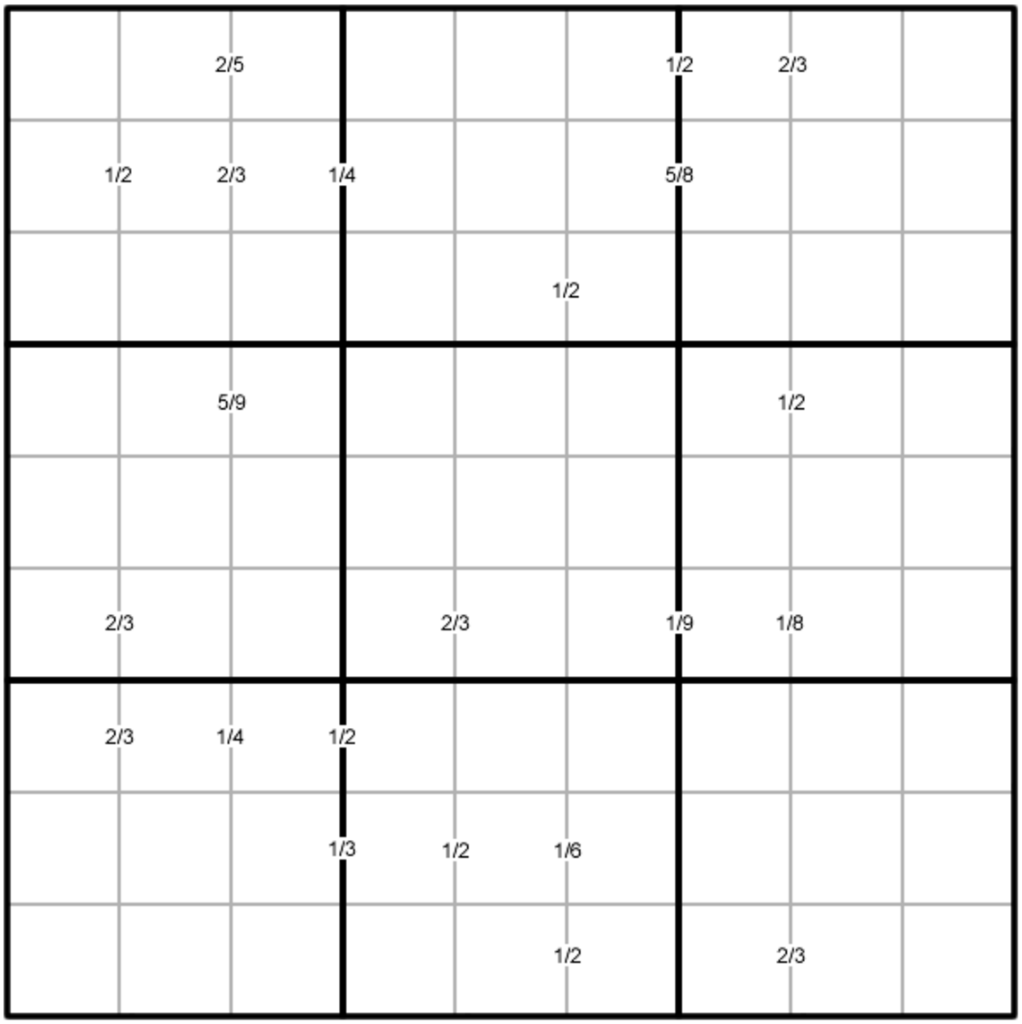
5) The area of a square frame is increased from 60 square inches to 86.4 square inches. By what percent were the side lengths increased?

6) The ratio of Jim's money to Peter's money was 4:7 at first. After Peter gave $\frac{3}{14}$ of his money to Jim, they have equal amounts of money. How much money did Jim have at first?

7) The ratio of John's allowance to Bill's allowance is 3:7. The ratio of John's allowance to Mary's allowance is 2:5. What is the ratio of Mary's allo wance to Bill's allowance?

8) Iniki has large, medium, and small metal bars. The large bars each weigh 8kg. The medium bars each weigh 6kg. The small bars each weigh 3kg. Iron, nickel, and lead are present in the ratio 4:1:3 in each large bar, 2:1:3 in each medium bar, and 1:1:1 in each small bar. If Iniki wants to melt together a combination of bars to make an allow that contains 40kg of iron, 20 kg of nickel, and 40kg of lead, how many small bars will she have to use?

Ratio Sudoku



Directions:

1) This is standard Sudoku with one exception: Each fraction is the ratio of the two numbers in the neighboring cells. For example, ½ can stand for the following combinations of numbers in the two neighboring cells: 1 and 2, 2 and 1, 2 and 4, 4 and 2, 3 and 6, 6 and 3, 4 and 8, 8 and 4.

2) Here are the standard rules for Sudoku: The object of the puzzle is to fill in the whole 9 x 9 grid with numbers 1 through 9 so that each horizontal line, each vertical line, and each of the nine 3 x3 squares (outlined with the bold lines) must contain all the nine different numbers 1 through 9.