Combinatorics Warm Up

1) How many four-letter "words" are there with vowels in the middle two places and consonants in the other two, and with no letter repeated?

2) In how many different ways can a student guess a complete set of answers to a five-item true/false quiz?

3) How many odd numbers with third digit 5 are there between 20,000 and 69,999 inclusive (digits can be repeated) ?

4) A yogurt shop has four different flavors and six different toppings. If a customer wanted to get one flavor and two different toppings, how many combinations could she get?

5) In a round-robin tournament, each of six softball teams plays each other team exactly once. How many softball games are needed?

6) (Optional Challenge: Skip if Necessary) We are given 5 distinct lines and two distinct circles in a plane. What is the maximum number of possible intersection points among these seven figures?

7) How many different ways are there to rearrange the letters in the word APPLE?

8) How many different ways are there to rearrange the letters in the word MINIMAL?

Combinatorics Warm Up (Solutions)

1) How many four-letter "words" are there with vowels in the middle two places and consonants in the other two, and with no letter repeated?

Example 25-4

How many four-letter "words" are there with vowels in the middle two places and consonants in the other two, and with no letter repeated?

Solution for Example 25-4: For the first and last letters, there are 21 and 20 choices. (Again, for the second of the two, 1 of the 21 possible choices is already off-limits.) For the middle two letters, there are 5 and 4 choices. The total is thus $\mathbf{21 \cdot 5 \cdot 4 \cdot 20}$. If we were doing this by filling in the blanks, we would write

$$\_ \quad \_ \quad \_ \quad \_ ,$$

then place a 21 in the first space, a 20 in the last, and a 5 and a 4 in the middle two. □

2) In how many different ways can a student guess a complete set of answers to a five-item true/false quiz?

Problem 457: Source: MATHCOUNTS 1985

In how many different ways can a student guess a complete set of answers to a five-item true/false quiz?

You may type any additional notes you have here.

Hide Solution

Reset

Your Submission: .

Solution: For each question there are two choices, so for all five there are $2^5 = \mathbf{32}$ choices.

3) How many odd numbers with third digit 5 are there between 20,000 and 69,999 inclusive?

Exercise 25-3:

How many odd numbers with third digit 5 are there between 20000 and 69999 inclusive?

You may type any additional notes you have here.

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Solution: Since the number must be odd, the last digit should be 1, 3, 5, 7, or 9, a total of 5 choices. The first digit must be 2, 3, 4, 5, or 6, for 5 choices. The second and fourth digits can be chosen in 10 ways each (anything from 0 to 9), and the third digit is already set. Writing the number of choices for each digit in that digit's position, we have

$$5 \cdot 10 \cdot 1 \cdot 10 \cdot 5 = \mathbf{2500}$$

such numbers.

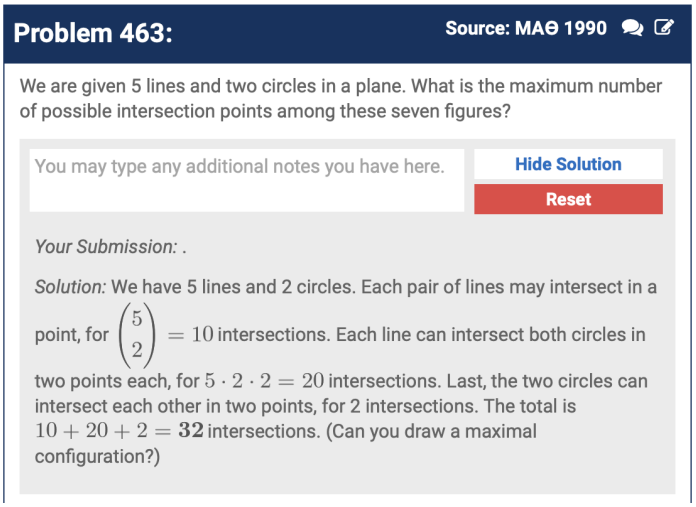
4) A yogurt shop has four different flavors and six different toppings. If a customer wanted to get one flavor and two different toppings, how many combinations could she get?

60 ways. (4 flavors)(6 x 5 / 2 toppings) = 60. Notice you have to divide the toppings by 2 because the order of the toppings doesn't matter. For example, getting chocolate chips *then* sprinkles is the same thing as getting sprinkles *then* chocolate chips. So if you don't divide 6 x 5 by 2, you'd end up double counting the toppings.

5) In a round-robin tournament, each of six softball teams plays each other team exactly once. How many softball games are needed?

15 games. Each of the 6 teams can be 5 other teams (so 6 x 5 = 30). But you have to divide by 2 to avoid double-counting the number of games.

6) (Optional Challenge: Skip if Necessary) We are given 5 distinct lines and two distinct circles in a plane. What is the maximum number of possible intersection points among these seven figures?



7) How many different ways are there to rearrange the letters in the word APPLE?

5 x 4 x 3 x 2 x 1 / 2 = 60 ways. This is because there are 6 letters to choose from for the first slot, 5 letters to choose from for the second slot, and so forth. You have to divide by 2 because there are two P's. If you don't divide, you'll end up double counting because a word might have the first P followed by the second P, and another copy of that word might have the second P followed by the first P.

8) How many different ways are there to rearrange the letters in the word MINIMAL?

7 x 6 x 5 x 4 x 3 x 2 x 1 / 2 / 2 = 1,260 ways. In this case, you have to divide by 2 twice since there are two M's and two I's.