Name: _____ Date: _____

AVERAGE RATE OF CHANGE
COMMON CORE ALGEBRA I

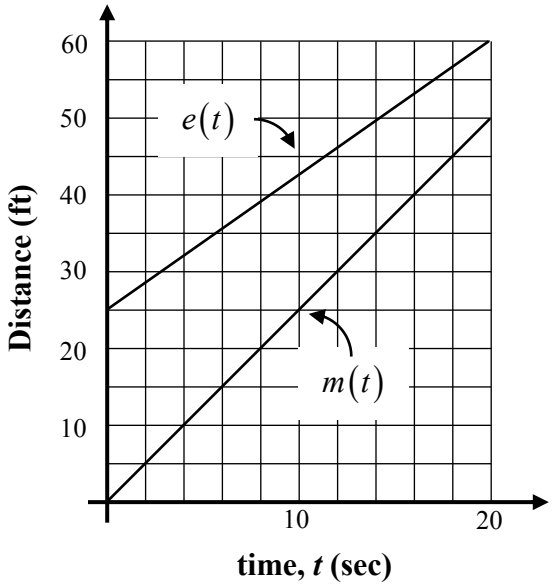
Functions are rules that give us **outputs** when we supply them with **inputs**. Very often, we want to know how **fast** the outputs are changing compared to a change in the input values. This is referred to as the **average rate of change** of a function.

***Exercise* #1:** Max and his younger sister Evie are having a race in the backyard. Max gives his sister a head start and they run for 20 seconds. The distance they are along in the race, in feet, is given below with Max's distance given by the function $m(t)$ and Evie's distance given by the function $e(t)$.

(a) How do you interpret the fact that $m(12) = 30$? Illustrate your response by using the graph.

(b) If both runners start at $t = 0$, how much of a head start does Max give his little sister? How can you tell?

(c) Does Max catch up to his sister? How can you tell?



(d) How far does Max run during the 20 second race? How far does Evie run? What calculation can you do to find Evie's distance?

(e) How fast do both Evie and Max travel? In other words, how many feet do each of them run per second? Express your answers as decimals and attach units.

MAX'S SPEED (FEET PER SECOND)	EVIE'S SPEED (FEET PER SECOND)

In the first exercise we were calculating the **rate** that the **function's output (y-values)** were changing compared to the **function's input (or x-values)**. This is known as finding the **average rate of change** of the function. You might think you've seen this before. And you have.

***Exercise* #2:** Finding the average rate of change is the same as finding the _____ of a line.

There is, of course, a formula for finding average rate of change. Let's get it out of the way.

AVERAGE RATE OF CHANGE

For the function $y = f(x)$, the average rate that $f(x)$ changes from $x = a$ to $x = b$ is given by:

$$\frac{f(b) - f(a)}{b - a} = \frac{\text{how much the y-values have changed}}{\text{how much the x-values have changed}}$$

***Exercise* #3:** Consider the function given by $f(x) = x^2 + 3$. Find its average rate of change from $x = -1$ to $x = 3$. Carefully show the work that leads to your final answer.

***Exercise* #4:** The function $h(x)$ is given in the table below. Which of the following gives its average rate of change over the interval $2 \le x \le 6$? Show the calculations that lead to your answer.

(1) $-\frac{3}{2}$

(2) $\frac{6}{4}$

(3) $-\frac{7}{6}$

(4) -1

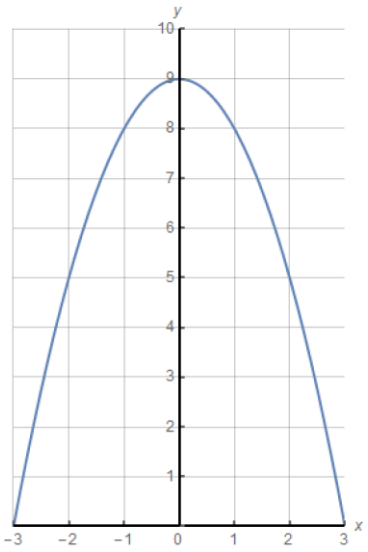
x	$h(x)$
0	10
2	9
4	6
6	3

***Exercise* #5:** Frances is selling glasses of lemonade. The function $g(t) = \frac{t^2 + 4}{2}$ models the number of glasses she has sold, *g*, after *t*-hours. What is the average rate at which she is selling lemonade between $t = 2$ and $t = 6$ hours. Include proper units in your answer.

Name: _____ Date: _____

AVERAGE RATE OF CHANGE

1. Consider the function given by $f(x) = 9 - x^2$. Find its average rate of change between the following points. Carefully show the work that leads to your final answer.



(a) $x = 0$ to $x = 3$ (b) $x = -1$ to $x = 5$

(c) $x = -2$ to $x = 2$

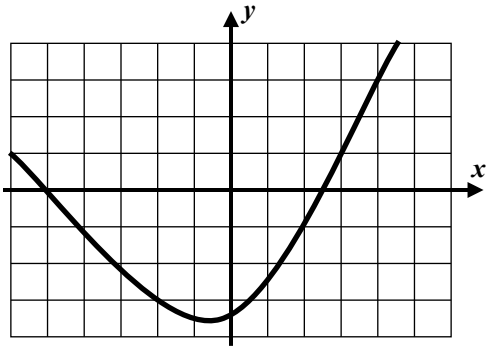
2. The function $f(x)$ is given in the table below. Find its average rate of change between the following points. Show the calculations that lead to your answer.

(a) $x = -3$ to $x = 1$ (b) $x = 0$ to $x = 4$.

x	$f(x)$
-3	7
0	-2
1	3
4	-8

3. The function $f(x)$ is given in the graph below. Find its average rate of change between the following points. Show the calculations that lead to your answer.

(a) $x = -6$ to $x = 4$ (b) $x = -2$ to $x = 2$.



APPLICATIONS

4. The following table shows the number of points the Arlington girls team scored in their last basketball game where t is the time passed in minutes and $f(t)$ the total number of points scored after t minutes.

t	$f(t)$
0	0
8	30
16	48
24	55
32	64

   (a) What was the average rate they were shooting in the first half of the game? Be sure to include proper units in your answer.

   (b) What was their average rate over the whole game?

   (c) Given your answers above which half of the game do you feel they had a better rate of scoring? Explain.

REASONING

5. Consider the function given by $f(x) = 6x + 5$.

   (a) Find its average rate of change from $x = 1$ to $x = 5$.

   (b) Find its average rate of change from $x = -2$ to $x = 6$.

   (c) The average rate of change for this function is always 6 (as you should have found in the first two parts of the problem). What type of function has a constant average rate of change? What do we call this average rate of change in this case? Search the Internet if needed.