

Problem of the Week: Are You Positive?

Part 1

Ms. Nomial's algebra students are playing Binomial Challenge. To start, each student makes up a first degree binomial. Then they split into groups of three. Finally, Ms. Nomial gives them a challenge to do with their group of binomials. Steve's group has these three:

$5 - 2x$

$x - 7$

$\frac{x}{2} + 3$

Today's challenge starts with writing an expression that consists of two of the binomials multiplied together and divided by the third binomial. They do not have to actually do the multiplication or division, they just need to write the expression. Then they have to find all values of x that will make the resulting value of their expression be positive.

What answer should Steve's group get for the values of x that make their expression positive?

Part 2

Steve's group is given the option to trade one of their binomials for one of the following other ones:

$\frac{2}{3}x - 8$

$1 - x$

$4x + 8$

The goal is for their new expression to have the smallest interval of positive x values that make the value of the expression positive. Which binomials should they swap? What positive x values will make their new expression be positive?

	Novice	Apprentice	Practitioner	Expert
Problem Solving				
Interpretation	Shows understanding of few of the criteria listed in the Practitioner column.	Shows understanding of most but not all of the criteria listed in the Practitioner column. For example, tries numbers and finds that the final result is always 2, but does not attempt to show why that happens.	Understands that: <ul style="list-style-type: none"> the overall sign of the expression is driven by the signs of the three individual binomials the placement of the binomials does not affect the overall sign of the expression in the binomials, three positives or two negatives and one positive result in an overall positive the goal of the problem is to determine for which values of x will the expression be positive 	Solves the main problem and the Extra correctly, and is at least a Practitioner in Strategy.
Strategy <i>(based on the solver's interpretation of the problem)</i>	Has no ideas that will lead them toward a successful solution. Has not written enough to tell what strategy they might have used.	Picks an incorrect strategy, or relies on luck to get the right answer. For example, uses Guess and Check, either for entire solution or to solve an equation or might simply try x values starting with negative values and move along the number.	Picks a sound strategy—success achieved through skill, not luck. Uses an appropriate strategy such as graphing the overall expression and to look for positive intervals or making sign charts for each binomial and using them to find the overall sign or writing.	Uses two separate strategies or one unusual or sophisticated strategy. For example, might do a case-by-case breakdown on the signs of the individual binomials, then include graphs.
Accuracy <i>(based on the chosen strategy)</i>	Has made many errors.	Has made several mistakes or misstatements.	Makes few or no mistakes of consequence and uses largely correct vocabulary.	[Generally not possible – can't be more accurate than Practitioner.]
Communication				
Completeness <i>(an incorrect solution can be complete)</i>	Has written very little that tells or shows how they found their answer.	Submitted explanation without work or work without explanation. Leaves out enough details that another student couldn't follow or learn from the explanation.	Defines variable(s). Shows equations, formulas, and calculations used and explains the rationale behind them. Shows the algebraic result of each step of the instructions and how they lead to the variable canceling and leaving only a value of 2 for the expression.	Adds in useful extensions and further explanation of some of the ideas involved The additions are helpful, not just "I'll say more to get more credit."
Clarity <i>(incomplete and incorrect solutions can be explained clearly)</i>	Explanation is very difficult to read and follow.	Explanation isn't entirely unclear, but would be hard for another student to understand. Explanation is long and is written entirely in one paragraph. Explanation contains many spelling and typing errors.	Explains the steps that they <i>do</i> explain in such a way that another student would understand (needn't be complete to be clear). Makes an effort to check formatting, spelling, and typing (a few errors are okay).	Format and organization make ideas exceptionally clear. Answer is very readable and appealing.