Field Trip Solutions

Solution: *42 students. Each paid $4.50*

Method 1: Make a Mathematical Model – cost per student

I started by choosing a variable:

Let n = the number of students in the sixth grade
n - 12 = the number of sixth graders without the baseball players

I know that the trip costs $189 and that the cost is shared equally among the students. That means the cost per student equals 189 divided by the number of students going. So originally, when the whole grade was going, the cost per student would have been 189/n dollars.

Once the baseball players couldn't go, only (n – 12) students went on the trip, so the cost per student would be 189/(n – 12) dollars. I know that the second cost is $1 more than the first one, so I wrote an equation expressing that:

$$(cost\ with\ n\ students) + 1 = (cost\ with\ n - 12\ students)$$
$$\frac{189}{n} + 1 = \frac{189}{n - 12}$$

I had two denominators with variables in them, so I multiplied both sides of the equation by (n)(n - 12) to cancel those denominators. I wound up with a quadratic equation, so I set it equal to 0.

$$\frac{n(n-12)}{1} \bullet \left(\frac{189}{n} + \frac{1}{1}\right) = \frac{189}{n-12} \bullet \frac{n(n-12)}{1}$$
$$189(n-12) + 1(n)(n-12) = 189n$$
$$198n - 2268 + n^2 - 12n = 1289n$$
$$n^2 - 12n - 2268 = 0$$

I factored the left side by finding factors of 2268 that differed by 12 (42 and 54). Since the product of the factors is 0, one or both of them must be 0, so I solved each factor to see what value of n would make it 0.

$$n^2 - 12n - 2269 = 0$$
$$(n-54)(n+42) = 0$$
$$n - 54 = 0\ or\ n + 42 = 0$$
$$n = 54\ or\ n = -42$$

The total number of sixth graders can't be negative, so I rejected the -42. That means there are 54 sixth graders. I divided 189 by 54 and found that each student would have paid $3.50 if they all went.

Without the twelve baseball players, only 54 - 12 or 42 students went. I divided 189 by 42 and found that each student paid $4.50. That checks since it's $1 more than $3.50, so the ticket price went up one dollar without the baseball players.

Method 2: System of Equations

I chose variables to represent the four unknown pieces of information in the problem:

Let n = the number of students in the sixth grade
c = the cost in dollars per student for the whole grade
n – 12 = the students who went on the trip without the players
c + 1 = the cost in dollars per student without the players

I know that the total cost of the trip is $189 and that equals the number of kids on the trip times how much each one pays. So I wrote two equations, one for the whole sixth grade going and one for after the baseball players couldn't go:

$$nc = 189$$
$$(n-12)(c+1) = 189$$

I distributed out the second equation and then substituted 189 for nc from the first equation and simplified:

$$(n-12)(c+1) = 189$$
$$nc + n - 12c - 12 = 189$$
$$189 + n - 12c - 12 = 189$$
$$n - 12c - 12 = 0$$

Note: From here there are a variety of different ways to solve the system of equations, including which variable to solve for first. I'll show some of them, and there are many more valid ones.

I solved my first equation for n and got $n = \frac{189}{c}$, then substituted that into the new equation so it had only c in it:

$$n - 12c - 12 = 0$$
$$\frac{189}{c} - 12c - 12 = 0$$

I multiplied the whole equation by c to cancel out the c under the 189. I found I had a quadratic equation so I set it equal to 0, then divided everything by 3 since all the numbers are multiples of 3:

$$c\left(\frac{189}{c} - 12c - 12\right) = c(0)$$
$$189 - 12c^2 - 12c = 0$$
$$0 = 12c^2 + 12c - 189$$
$$0 = 4c^2 + 4c - 63$$

I factored the right side and then solved each factor to see what value of c would make it 0 since their product has to be 0 and that only happens when at least one of the factors is 0:

$$0 = 4c^2 + 4c - 63$$
$$0 = (2c-7)(2c+9)$$
$$2c - 7 = 0 \text{ or } 2c + 9 = 0$$
$$c = \frac{7}{2} \text{ or } c = -\frac{9}{2}$$

It doesn't make sense for the cost per student to be negative, so I rejected the –29 answer. That means the cost for each student was $\frac{7}{2}$ dollars or $3.50. Substituting that into the equation nc = 189, I solved for n:

$$nc = 189$$
$$3.5n = 189$$
$$n = 54$$

There are 54 sixth graders and the original plan was for them to pay $3.50 each. When the 12 baseball players couldn't go, the 54 - 12 or 42 remaining students paid 3.50 + 1 or $4.50 each. To check, I multiplied 42 times 4.50 to see if it made 189, and it did.