Buried Treasure Solutions

Solution: *The x-coordinate where Callie should dig is 15*.

Method: Create a System of Equations

Part 1
First, let's create the line that is "exactly northwest" of the waterfall. "Exactly northwest" means the line must have a slope of -1. In addition, it passes through the waterfall, so the line contains the coordinates (26, 2). The equation of a line is $y = mx + b$. We know the slope but we don't know the y-intercept. So let's make a table:

x	y
0	?
26	2

I reason that because the slope is -1, then every time you decrease x by 1, you *increase* y by 1. Thus, the y-intercept is 2 + 26 = 28.

So the equation of the line is $y = -x + 28$.

Part 2
Next, let's create the line that "starts halfway between the rock and the old stump". The point that is halfway between the rock and the stump has an x-coordinate halfway between 1 and 5 and a y-coordinate halfway between 11 and 3 (*can you figure out why this makes sense?*). So the halfway point is (3, 7).

Since we "walk so that the distance between ye and the stump is always the same as the distance between ye and the rock", we need to draw the line so that it's always angled exactly halfway between the tree and the rock. If you plot this carefully on a graph, you'll find that for this to happen, the slope needs to be equal to $\frac{1}{2}$. **Next year**, you'll learn that the $\frac{1}{2}$ comes from the fact that the slope between the stump and the rock is $-\frac{2}{1}$, and so to rotate the line 90 degrees, you need to change the sign and flip the fraction. This is called using the *negative reciprocal* to create a *perpendicular line*.

Coming back to the problem, so the line passes through (3, 7) and has a slope of $\frac{1}{2}$. Using the same table-logic above, the y-intercept is at $\frac{11}{2}$.

So the equation of the line is $y = \frac{1}{2}x + \frac{11}{2}$.

Part 3
Solve this system of equations (using Desmos and/or an algebraic method):

$$x = 15$$
$$y = 13$$