Lettuce for Sale Solutions

Solution: *48 heads of lettuce were sold each day*

Method 1: Make a Mathematical Model: Working Forwards

I made a table to keep track of the six different times in the problem and how many heads of lettuce were in the case at each time. I knew that on Monday morning there were 80 heads, so I started with that number. I know that he sold the same number of heads each day, so I chose a variable:

Let x = the number of heads of lettuce sold each day

Here's the table I made. I'll explain it underneath:

Day	Monday	Tuesday	Wednesday
Morning	80	2(80 – x) or 160 – 2x	3(160 – 3x) or 480 – 9x
Evening	80 - x	160 – 2x – x or 160 – 3x	480 – 9x – x or 480 – 10x

Following the problem, if he started with 80 on Monday and sold x, there would be 80 – x left on Monday night. Then, that amount was doubled for Tuesday morning, so there were 2(80 – x) on Tuesday morning. I simplified that to 160 – 2x. Since he sold x again on Tuesday, at the end of the day there were 160 – 2x – x or 160 – 3x left in the case. He tripled that amount to start on Wednesday, so there were 3(160 – 3x) or 480 – 9x in the case. He again sold x on Wednesday, leaving 480 – 9x – x or 480 – 10x in the case at the end of the day.

Since I know there were 0 left at the end of Wednesday, I wrote an equation and solved it for x:

$$480 - 10x = 0$$
$$480 = 10x$$
$$48 = x$$

He sold 48 heads of lettuce each day.

Method 2: Working Backwards

I made a table to keep track of the six different times in the problem and how many heads of lettuce were in the case at each time. I knew that on Wednesday evening there were 0 heads left, so I started with that number. I knew that the amount in the case on Wednesday morning was three times what was left on Tuesday evening, so I picked a variable:

Let x = the number of heads of lettuce left on Tuesday evening

That meant that there would be 3x heads of lettuce in the case on Wednesday morning. And since he sold all of that on Wednesday, he must have sold 3x on Wednesday. He sold the same amount each day, so he also sold 3x on Tuesday and on Monday, so:

Let 3x = the number of heads of lettuce sold each day

Here's the table I made. I'll explain it underneath:

Day	Monday	Tuesday	Wednesday
Morning	2x + 3x or 5x	x + 3x or 4x	3x
Evening	4x/2 or 2x	x	0

I've explained the x and 3x already, so starting with the x on Tuesday evening, if he sold 3x on Tuesday there must have been (x + 3x) or 4x in the case on Tuesday morning. Since Tuesday morning was double the amount left on Monday evening, Monday evening must be half of Tuesday morning, so there were $\frac{4x}{2}$ or 2x on Monday evening. And since he also sold 3x on Monday, there must have been (2x + 3x) or 5x on Monday morning.

Since I know there were 80 heads of lettuce in the case on Monday morning, I wrote an equation and solved it for x:

$$45x = 80$$
$$x = 16$$

The number of heads of lettuce sold each day was 3x, so it's 3(16) or 48.

He sold 48 heads of lettuce each day.

Method 3: Mix of Forwards and Backwards

This solution path is sort of a hybrid of the two above, where the general idea is to start at each end and meet somewhere in the middle. There might be various meeting points, but most likely it would be Tuesday morning or evening. Here's one example of what it might look like.

I made a table to keep track of the six different times in the problem and how many heads of lettuce were in the case at each time. I know that he sold the same number of heads of lettuce each day, so I chose a variable:

Let x = the number of heads of lettuce sold each day

Here's the table I came up with. I'll explain it underneath:

Day	Monday	Tuesday	Wednesday
Morning	80	2(80 – x) or 160 – 2x	x
Evening	80 - x	$\frac{x}{3}$	0

I knew there were 80 heads on Monday morning, and x were sold, so there were 80 – x left in the case on Monday evening. That amount was doubled on Tuesday morning, so there were 2(80 – x) or (160 – 2x) on Tuesday morning. I also knew there were 0 left on Wednesday evening, so if x were sold on Wednesday, there must have been x in the case on Wednesday morning. Wednesday morning was three times what was left Tuesday evening, so Tuesday evening would be one-third of Wednesday morning, or $\frac{x}{3}$.

Now I have expressions for Tuesday morning and Tuesday evening, and I know that x more were sold on Tuesday, so I could wrote an equation that Tuesday morning – x = Tuesday evening:

$$\begin{aligned} 160 - 2x - x &= \frac{x}{3} \\ 160 - 3x &= \frac{x}{3} \\ 480 - 9x &= x \\ 480 &= 10x \\ 48 &= x \end{aligned}$$

He sold 48 heads of lettuce each day.

Method 4: Make a Mathematical Model: One Big Equation

The same number of heads of lettuce were sold each day, so chose a variable to represent that number:

Let x = the number of heads of lettuce sold each day

There were 80 heads of lettuce on Monday morning, so after x were sold that day, there were 80 – x left. On Tuesday morning, he doubled the amount from Monday night, so there were 2(80 – x). Again x were sold on Tuesday, so by Tuesday night there were 2(80 – x) – x in the case. That amount was tripled on Wednesday morning, so there were 3[2(80 – x) – x] at that point. Again x were sold during the day, so by Wednesday night there were 3[2(80 – x) – x] – x heads of lettuce in the case. I know that there were 0 left on Wednesday night, so I wrote an equation:

$$\begin{aligned} 3[2(80 - x) - x] - x &= 0 \\ 3[160 - 2x - x] - x &= 0 \\ 480 - 6x - 3x - x &= 0 \\ 480 - 10x &= 0 \\ 480 &= 10x \\ 48 &= x \end{aligned}$$

He sold 48 heads of lettuce each day.