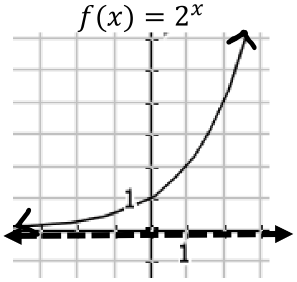
Unit 4 - Exponential Functions - Study Guide

Parent Function

$f(x) = 2^x$



domain: $-\infty < x < \infty$
range: $y > 0$
y-intercept: (0,1)
zeros: none
asymptote: y = 0
increasing: $-\infty < x < \infty$

Evaluating Exponential Functions

EXAMPLE: If $f(x) = 20\left(\frac{1}{2}\right)^x$ find $f(2)$.

→SOLUTION: $f(2) = 20\left(\frac{1}{2}\right)^2 = 20\left(\frac{1}{4}\right) = 5$

So… $f(2) = 5$

…which means $f(x)$ passes through the point (2,5).

Linear versus Exponential

Linear

Look for **addition** or **subtraction** of y-values

+1 +1 +1 +1

x	0	1	2	3	4
y	8	5	2	-1	-4

-3 -3 -3 -3

- Has a constant rate of change:
  Slope $= \frac{\text{y change}}{\text{x change}} = \frac{-3}{1} = -3$
- Has a y-intercept at (0, 8)
- Equation: $y = mx + b$
  m = slope and b = y-intercept
  $y = -3x + 8$

Exponential

Look for **multiplication** or **division** of y-values

+1 +1 +1 +1

x	0	1	2	3	4
y	2	6	18	54	162

•3 •3 •3 •3

- Does NOT have a constant rate of change
- Has a growth factor of 3
- Has y-intercept at (0, 2)
- Equation: $y = a(b)^x$
  a = initial value and b = growth factor
  $y = 2(3)^x$

Average Rate of Change

- NEED 2 POINTS
- The SLOPE of a line that passes through two points of a function
- $\frac{y\ change}{x\ change}$

EXAMPLE: Given the function $f(x) = 3(2)^x$, find the average rate of change over the interval $2 \le x \le 4$.

→SOLUTION:

x	f(x)
2	12
4	48

+2 +36

$f(2) = 3(2)^2 = 12$

$f(4) = 3(2)^4 = 48$

a.r.o.c. $= \frac{36}{2} = 18$

Laws of Exponents

- Multiplying with the same base: ADD powers
  $x^a \bullet x^b = x^{a+b}$ EXAMPLE: $5^3 \bullet 5^7 = 5^{10}$
- Dividing with the same base: SUBTRACT powers
  $\frac{x^a}{x^b} = x^{a-b}$ EXAMPLE: $\frac{x^9}{x^5} = x^4$
- Power to power: MULTIPLY powers
  $(x^a)^b = x^{a \cdot b}$ EXAMPLE: $(2^3)^5 = 2^{15}$
- Anything to the FIRST power is ITSELF
  $x^1 = x$ EXAMPLE: $20^1 = 20$
- Anything to the ZERO power is ONE
  $x^0 = 1$ EXAMPLE: $20^0 = 1$
- Negative exponents: re-write as $\frac{1}{}$ the exponent to the positive power (it is not a negative number!!!)
  $b^{-n} = \frac{1}{b^n}$ EXAMPLE: $2^{-4} = \frac{1}{2^4} = \frac{1}{16}$

Translating Exponential Functions

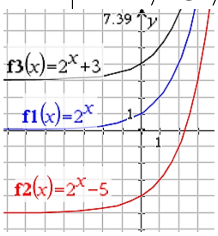
$f(x) = 2^x$ has a y-intercept at (0,1) and asymptote at y = 0 !!!!!!

The number OUTSIDE the exponent moves the parent function UP + or DOWN –

EXAMPLES:

$y = 2^x - 5$ down 5 asy. @ y = –5

$y = 2^x + 3$ up 3 asy. @ y = 3



*the constant is the asymptote

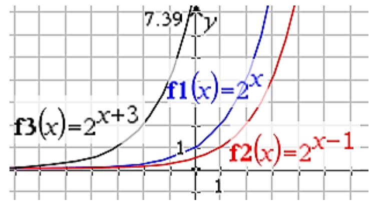
The number INSIDE the power moves the parent function LEFT + or RIGHT –

*** *opposite of what you think!*

EXAMPLES:

$y = 2^{x-1}$ right 1

$y = 2^{x+3}$ left 3



*asymptote does not change

The BASE tells you if the function is

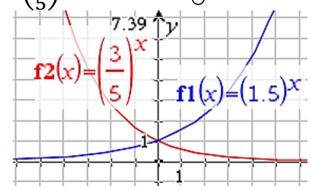
INCREASING if $b > 1$

DECREASING if $0 < b < 1$

EXAMPLES:

$y = 1.5^x$ increasing

$y = \left(\frac{3}{5}\right)^x$ decreasing



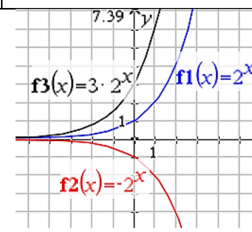
The number IN FRONT of the exponent

- **reflects** the function over the x–axis if –
- the value changes the **y–int.** (must be $y = a(b)^x$ form)

EXAMPLES:

$y = -2^x$ reflects over x–axis

$y = 3(2)^x$ y–intercept at 3



Exponential Growth and Decay

Growth: ADD	Decay: SUBTRACT
$y = a(1 + r)^t$	$y = a(1 - r)^t$

a = initial amount

r = growth/decay rate as decimal

t = time

GROWTH EXAMPLE:

A population of bugs is growing at a rate of 5% per day. The initial population is 22 bugs. Find a formula that models this situation.

→SOLUTION:

100% + 5% = 105% as a decimal 1.05

So… $y = 22(1.05)^x$

DECAY EXAMPLE:

A radioactive material that is initially 55 grams decays at a rate of 14% per day. Find a formula that models this situation.

→SOLUTION:

100% – 14% = 86% as a decimal is 0.86

So… $y = 55(0.86)^x$

Geometric Sequences

Sequence means MAKE A TABLE

$$a_n = a_1 r^{n-1}$$

a_n = the n^{th} term/any term

n = term number

a_1 = initial value

r = common ratio (growth/decay factor)

EXAMPLE:

Write a formula that can be used to find the n^th term of the sequence:

20, 10, 5, 2.5, …

Then find the 15th term.

→SOLUTION:

$f(10) = 40\left(\frac{1}{2}\right)^{10}$

$= 0.0390625$

term	0	1	2	3	4
#	40	20	10	5	2.5

$\cdot\frac{1}{2}$ $\cdot\frac{1}{2}$ $\cdot\frac{1}{2}$ $\cdot\frac{1}{2}$

$f(n) = 40\left(\frac{1}{2}\right)^n$

Percents Tips

- convert to a decimal →move decimal 2 units left
- Increase means ADD to 100%
- Decrease means SUBTRACT from 100%