

Unit 5 - Polynomials - Study Guide



Important Vocabulary

Term – A number, variable or product of numbers and variables

Like/similar terms – Two or more terms that have the same variable(s) with the same exponents

Unlike terms – Two or more terms with different variables and/or different powers

Monomial – A single variable or number, or a product of a coefficient and one or more variables with exponents that are whole numbers

Binomial – A polynomial with two terms

Trinomial – A polynomial with three terms

Simplest form – When a polynomial contains no like terms

Standard form – When a polynomial is written with the exponents in descending order

Coefficient – The number being multiplied to the variable (written in front of variable)

Leading coefficient – The coefficient of the variable with the highest degree

Constant – A number that is not attached to any variable

Variable – A letter or symbol used to replace a number

Degree of a polynomial – the degree of the monomial with the greatest exponent

Polynomial

An expression made up of variables and constants.
The exponents are always WHOLE NUMBERS!!!

Adding/Subtracting Polynomials

1. Distribute
2. Combine like terms
3. Write your answer in standard form

Tips:

- Powers NEVER change
- Write in coefficients of 1
EX: $2x + x = 2x + 1x = 3x$, not $2x$
- Be careful when distributing a negative
EX: $(3x + 1) - (2x - 4) = 3x + 1 - 2x + 4$
- Subtract x from y looks like $y - x$
- SUM or TOTAL means ADD
- DIFFERENCE means SUBTRACT
- PERIMETER means add up ALL of the sides

Multiplying Polynomials

1. Write in exponents and coefficients of 1
2. Distribute (if necessary)
3. Multiply coefficients
4. Multiply variables by ADDING powers
(or if power to a power MULTIPLY)

Tips:

- Use the LAWS OF EXPONENTS
- Powers ALMOST ALWAYS change
- PRODUCT means MULTIPLY
- Be careful of integers
- Pay attention to what type of polynomials you start with
(never end with more terms than your largest polynomial)
EX: $(4x^4y^3z^3)(2x^2y^2z) = 8x^6y^5z^4$
(mono*mono = mono)
- Expand whenever you see a power
EX: $(5x)^3 = 5x \bullet 5x \bullet 5x = 125x^3$
EX: $(x + 2)^2 = (x + 2)(x + 2)$
- AREA and VOLUME means multiply

Laws of Exponents

- ⇒ Anything to the FIRST power is ITSELF
 $20^1 = 20$
- ⇒ Anything to the ZERO power is ONE
 $20^0 = 1$
- ⇒ When MULTIPLYING exponents that have the same base, ADD the powers
 $x^2 \bullet x^4 = x^6$
- ⇒ When DIVIDING exponents that have the same base, SUBTRACT the powers
 $x^{10} \div x^8 = x^2$
- ⇒ When raising an exponent to another power, MULTIPLY the powers.
 $(x^2)^4 = x^8$

Adding vs. Multiplying

Simplify: $3x^2 + 5x^2$
 $= 8x^2$

Add coefficients
Powers don't change

Simplify: $3x^2 \bullet 5x^2$
 $= 15x^4$

Multiply coefficients
Add powers

Double Distributing (and beyond)

Be careful of INTEGER RULES!

(binomial)(binomial)	(binomial) ²	(binomial)(trinomial)
$(x+3)(x-7)$ $x(x-7) + 3(x-7)$ $x^2 - 7x + 3x - 21$ $x^2 - 4x - 21$	$(x-5)^2$ $(x-5)(x-5)$ $x(x-5) - 5(x-5)$ $x^2 - 5x - 5x + 25$ $x^2 - 10x + 25$	$(x-1)(x^2+3x-2)$ $x(x^2+3x-2) - 1(x^2+3x-2)$ $x^3 + 3x^2 - 2x - x^2 - 3x + 2$ $x^3 + 2x^2 - 5x + 2$

Completing the Square

Example:

$$x^2 - 8x + 22$$

$$x^2 - 8x + 22$$

$$(x-4)(x-4) + 22 - 16$$

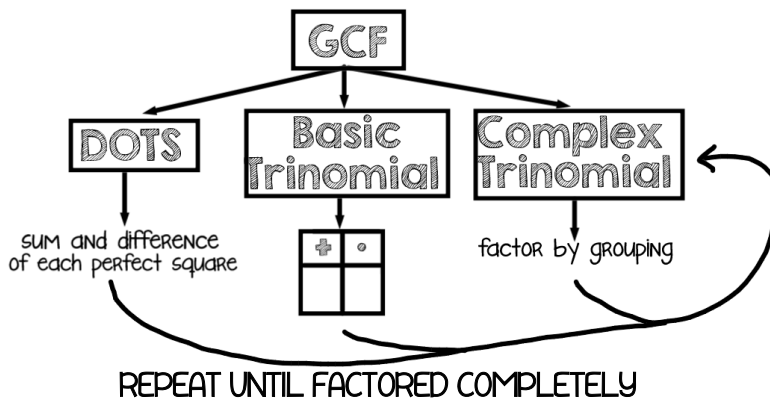
$$-4 \quad x-4 = 16$$

$$(x-4)^2 + 6$$

$x^2 - 8x + 22$
is the same as
 $(x-4)^2 + 6$

Factoring Completely

- Remember: we always RE-WRITE and NEVER change the value of an expression
- Always check by multiplying back through to make sure you get what you started with (or use calculator magic number technique)



Examples:

$\begin{array}{r} + \cdot \\ \hline 4 \overline{) -45} \\ \underline{9} \\ 9 - 45 \\ \hline \end{array}$ $9 \div -5 = 4 \checkmark$ $9 \cdot -5 = -45 \checkmark$	$x^2 + 4x - 45$ $(x+9)(x-5)$	$\begin{array}{r} + \cdot \\ \hline 7 \overline{) 10} \\ \underline{5} \\ 5 - 10 \\ \hline \end{array}$ $5 \div 2 = 7 \checkmark$ $5 \cdot 2 = 10 \checkmark$	$x^4 + 7x^2 + 10$ $(x^2+5)(x^2+2)$
\star DOTS $b \cdot b = b^2 \checkmark$ $5 \cdot 5 = 25 \checkmark$ subtract \checkmark	$b^2 - 25$ $(b+5)(b-5)$	GCF: $2m$ $2m(m^2 - 13m + 36)$ $\begin{array}{r} + \cdot \\ \hline 13 \overline{) 36} \\ \underline{-9} \\ -9 - 36 \\ \hline \end{array}$ $-9 \div -4 = -13 \checkmark$ $-9 \cdot -4 = 36 \checkmark$	$2m^3 - 26m^2 + 72m$ $2m(m^2 - 13m + 36)$ $2m(m-9)(m-4)$
GCF: 3 $\begin{array}{r} + \cdot \\ \hline 11 \overline{) 18} \\ \underline{-9} \\ -9 - 18 \\ \hline \end{array}$ $-9 \div -2 = 18 \checkmark$ $-9 \cdot -2 = 18 \checkmark$	$18a^2 - 33a + 9$ $3(6a^2 - 11a + 3)$ $3[3a(2a-3) - 1(2a-3)]$ $3(3a-1)(2a-3)$	DOTS $x^2 \cdot x^2 = x^4 \checkmark$ $9 \cdot 9 = 81 \checkmark$ DOTS $x \cdot x = x^2 \checkmark$ $3 \cdot 3 = 9 \checkmark$	$x^4 - 81$ $(x^2+9)(x^2-9)$ $(x^2+9)(x+3)(x-3)$

Closure

A set is CLOSED under an operation if it produces a member of that same set. Polynomials are closed under addition, subtraction, multiplication but NOT division.

EXAMPLES:

Addition: $3x^2 + 8x^2 = 11x^2$

Subtraction:

$$3x^2 - 8x^2 = -5x^2$$

Multiplication:

$$3x^2 \cdot 8x^2 = 24x^4$$

NON-EXAMPLE:

Division: $\frac{10x^3}{5x^8} = 2x^{-5}$ ***NOT a polynomial